Section 11.9 Solutions math 31

4. $f(x) = \frac{5}{1-4x^2} = 5\left[\sum_{n=0}^{\infty}(4x^2)^n\right] = 5\sum_{n=0}^{\infty} 4^n x^{2n}$ when $|4x^2|<1$ or $R=\frac{1}{2}$ Interval of Convergence: $-\frac{1}{2}<x<\frac{1}{2}$

6. $f(x) = \frac{1}{x+10} = \frac{1}{10}\left(\frac{1}{1-(-\frac{x}{10})}\right) = \frac{1}{10}\sum_{n=0}^{\infty}\left(-\frac{x}{10}\right)^n = \sum_{n=0}^{\infty}\frac{(-1)^n x^n}{10^{n+1}}$

when $\left|-\frac{x}{10}\right|<1$ or $R=10$ Interval of Convergence: $-10<x<10$

8. $f(x) = \frac{x}{2x^2+1} = x\left[\frac{1}{1-(-2x^2)}\right] = x\sum_{n=0}^{\infty}(-2x^2)^n = \sum_{n=0}^{\infty}(-1)^n 2^n x^{2n+1}$

when $|-2x^2|<1$ or $R=\frac{1}{\sqrt{2}}$ Interval: $-\frac{1}{\sqrt{2}}<x<\frac{1}{\sqrt{2}}$

26. $\frac{t}{1+t^3} = t\left[\frac{1}{1-(-t^3)}\right] = t\sum_{n=0}^{\infty}(-t^3)^n = \sum_{n=0}^{\infty}(-1)^n t^{3n+1}$

$\therefore \int \frac{t}{1+t^3}\,dt = C + \sum_{n=0}^{\infty}\frac{(-1)^n t^{3n+2}}{3n+2}$, $R=1$

28. $\frac{\tan^{-1}x}{x} = \frac{1}{x}\left[\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{2n+1}\right] = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{2n+1}$

$\therefore \int \frac{\tan^{-1}x}{x}\,dx = C + \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)^2}$, $R=1$